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Libration Control of Tethered Satellites in Elliptical Orbits

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Introduction

TETHERED satellite systems can have important applications in many future space missions. A significant amount of research has been conducted into the dynamics and control of tethered systems for applications such as electrodynamic orbit control, upper atmospheric research, payload transfer, and so on. It is known that for systems in circular orbits the tether possesses a stable equilibrium configuration along the orbit local vertical.[1] This is the nominal stationkeeping configuration for such systems, and deployment and retrieval control is almost exclusively related to this system arrangement. In more recent times, interest in utilizing tethered systems in elliptical orbits for the purposes of momentum transfer,[2,3] atmospheric observation,[4] or communications[5] has generated research on both electrodynamic and nonelectrodynamic tethers in noncircular orbits. In some of these applications, it is necessary to restrict the librational motion of the tether and to prevent tumbling or rotation of the system.

Schechter[6] first studied the librations of a fixed-length dumbbell satellite on elliptic orbits using a power series solution of the libration equation. Swan[7] studied the in-plane dynamics and control of tethers in elliptic orbits and showed that the libration dynamics are driven by the magnitude of the orbit eccentricity. He found that the system begins tumbling at an eccentricity of $e = 0.355$. Control via tether reeling using a proportional-derivative control scheme was considered to keep the tether pointing along the local vertical for $e < 0.355$ and to prevent tumbling for $e > 0.355$. Fujii and Ichiki[8] studied the effects of orbit eccentricity and tether elasticity on the behavior of tethered systems during stationkeeping. The dynamics were studied using Poincaré maps, bifurcation diagrams, and Lyapunov exponents. They showed the periodic, quasi-periodic, and chaotic motion of the system with respect to the orbit eccentricity. Naigang et al.[9] studied the in-plane stability of the deployment and retrieval processes of a tethered satellite in elliptic orbits and suggested that there are certain conditions under which deployment and retrieval can be accomplished in an equilibrium condition with respect to libration. This has also been demonstrated by Beletsky and Levin.[10] Ruiz et al.[11] studied the stability of a tethered system in elliptic orbits without a subsatellite attached at one end. Certain instabilities caused by resonance or low tension were observed. Takeichi et al.[4,12] studied the three-dimensional librations of tethered systems in elliptic orbits under the influence of atmospheric drag for single[12] and multiple subsatellites.[4] It was shown that instability of the system can increase as the length of tether increases because of greater atmospheric drag on the tether(s) and subsatellite(s). Misra et al.[13] considered the dynamics of a constant-length tethered satellite system. They studied the in- and out-of-plane librational dynamics using nonlinear numerical methods and found quasi-periodic solutions for small initial conditions and low orbit eccentricities and also showed that increasing the orbit eccentricity leads to chaotic motion.

The nature and stability of the librations of general gravity-gradient-oriented satellites were studied by Modi and Brereton.[14,15] The stability was first investigated in Ref. 16 using concepts of integral manifolds in a phase space, which provides the limiting invariant surface of librational states for which the satellite remains stable. A surface in the phase space with zero cross section corresponds to a periodic solution, and increasing the orbit eccentricity causes the invariant surface to shrink so that only periodic solutions are possible at the critical eccentricity for stable motion. The stability of periodic solutions was obtained using Floquet theory in Ref. 15, which involves a linear perturbation analysis of the solutions.

Libration control of tethered satellites in elliptic orbits has been considered by Fujii et al.[17] using time-delayed feedback control, assumed to be implemented via thrusters. Takeichi et al.[18] also studied the control of the librational motion of a tethered satellite system following deployment in an elliptical orbit. Using thrusters, they showed that the system can be controlled into a periodic solution. Kojima et al.[19] considered the control of a three mass constant-length tethered system in low eccentricity elliptic orbits via thrusters using delayed feedback control and decoupled control. Although methods such as delayed feedback control can be made robust with respect to parameter variations, its application using thrusters is not desirable because it can consume significant amounts of propellant over long periods of time.

This Note presents an approach for controlling the librational motion of tethered satellite systems in elliptic orbits using only forced length variations. The novelty of the approach is that the reference trajectory for the system is generated as a controlled periodic solution to the equations of motion. In particular, the tether tension and length dynamics are explicitly accounted for. The stability of the solutions is analyzed using Floquet theory, and it is shown that the periodic solution can be stabilized by a linear receding horizon tracking controller whose feedback gains are 2π-periodic coefficients.

Tether System Model

For simplicity, only the in-plane motion of the system is treated in this Note. The out-of-plane motion is unstable in elliptical orbits and is generally undesirable.[10] Although the concepts developed in this Note can be applied to the case of coupled in- and out-of-plane librations, it is known that control of the out-of-plane librations using length variations is slow unless large variations in length are used.[1] Hence, another control strategy such as thrusters might need to be considered to stabilize the out-of-plane dynamics. In this case, the out-of-plane dynamics could be considered as a perturbation to the in-plane dynamics.

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The equations of motion for a straight, massless inelastic tether in an elliptic orbit are given by[10]

$$\theta'' = 2(\theta' + 1)\left[\frac{e\sin\nu}{(1 + e\cos\nu)} - \Lambda'/\Lambda\right] - \frac{3\sin\theta\cos\theta}{(1 + e\cos\nu)} \quad (1)$$

$$\Lambda'' = \frac{2e\Lambda'\sin\nu}{(1 + e\cos\nu)} + \Lambda\left[(\theta' + 1)^2 + \frac{(3\cos^2\theta - 1)}{(1 + e\cos\nu)}\right] - \bar{T} \quad (2)$$

where θ is the in-plane libration angle, e is the orbit eccentricity, ν is the true anomaly, Λ is the nondimensional tether length, $\bar{T} = T/(m_s\omega^2 L)$ is the nondimensional tether tension, T is the tether tension, m_s is the subsatellite mass, L is the tether length, and $()' = \mathrm{d}()/\mathrm{d}\nu$ is the nondimensional time derivative.

Typically, control of the system is achieved via manipulating the tether tension $\bar{T}$ in Eq. (2). However, for more robust control design the control input is selected as the reel acceleration $u = \Lambda''$, and Eq. (2) is utilized as a nonlinear path constraint in the problem formulation by enforcing

$$\bar{T}_{\min} \le 2e\Lambda'\frac{\sin\nu}{(1 + e\cos\nu)} + \Lambda\left[(\theta' + 1)^2 + \frac{(3\cos^2\theta - 1)}{(1 + e\cos\nu)}\right] - u \le \bar{T}_{\max} \quad (3)$$

along the trajectory, where $\bar{T}_{\min} = 0.001$ and $\bar{T}_{\max} = 100$ are selected to keep the open-loop solution physically realizable. In most studies concerning the dynamics of tethered satellites in elliptic orbits, almost no mention is made of the constraint imposed by the tether tension. The strategy used for designing the control system takes into consideration this constraint when formulating the open-loop controls.

Closed-Loop Controlled Periodic Trajectories

Periodic solutions for tethered systems in elliptic orbits have been computed both analytically and numerically in previous work. Examples of numerical solutions using continuation of periodic trajectories are given in Ref. 20. In this approach, the analytical equations representing the system Jacobian with respect to the states and continuation parameter must be integrated over the period of the desired orbit. By slowly changing the continuation parameter, families of periodic solutions can be generated with a predictor-corrector strategy. This approach works well where no control is required. However, if manipulation of the tether length is desired to augment the periodic solution, then another approach is needed.

Computation of Periodic Trajectories

Consider the general nonlinear system governed by the state equations

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t; p) \quad (4)$$

where $\boldsymbol{x} \in \mathbb{R}^{n_x}$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^{n_u}$ is the vector of control inputs, t is the time, and p is a parameter governing the nature of the trajectory. A periodic trajectory and possibly a corresponding period T_p are sought such that

$$\boldsymbol{x}(t) = \boldsymbol{x}(t + T_p), \qquad \boldsymbol{u}(t) = \boldsymbol{u}(t + T_p) \quad (5)$$

In the case where pure periodic trajectories are desired, that is, no additional active control, then Eq. (5) is still satisfied if

$$\boldsymbol{u}(t) = \boldsymbol{u}_0(t) \quad (6)$$

where $\boldsymbol{u}_0 \in \mathbb{R}^{n_u}$ are possible nonzero control inputs that are T_p periodic. For most satellites controlled using thrusters, $\boldsymbol{u}_0 \equiv 0$ is typical. However, an example where a nonzero control input is present is when the tethered satellite system is controlled using tension control. In this case, the fixed-length tether solution has a nominal nonzero tension control input that must be considered when using Eqs. (1) and (2) to design the controller. In reality, the tether reel would be locked, and the nonzero tether tension would be caused by natural variations from the gravity-gradient and other effects, rather than from the control system.

In addition to satisyfing Eq. (5), it might be desirable to place some general restrictions on the motion, such that

$$\boldsymbol{g}_L \le \boldsymbol{g}[\boldsymbol{x}(t), \boldsymbol{u}(t), t; p] \le \boldsymbol{g}_U \quad (7)$$

where $\boldsymbol{g} \in \mathbb{R}^{n_g}$ are general nonlinear path constraints. Finally, it is generally the case that a zero-control solution is sought when possible, and so the following cost functional should be minimized [the cost function defined in Eq. (8) could, in principle, be selected as any cost function, not merely a weighted norm]:

$$\mathcal{J} = \int_t^{t+T_p} \left\|\boldsymbol{w}_u^T(\boldsymbol{u} - \boldsymbol{u}_0)\right\|_q \mathrm{d}t^* \quad (8)$$

where $\boldsymbol{w}_u \in \mathbb{R}_+^{n_u}$ is a vector of weighting coefficients and $\|\boldsymbol{h}(t)\|_q$ is the l_q norm of $\boldsymbol{h}(t)$ defined by

$$\|\boldsymbol{h}(t)\|_q = \left[\sum_{i=1}^{m} |h_i(t)|^q\right]^{1/q}, \qquad 1 \le q \le \infty \quad (9)$$

Instead of treating the problem defined by Eqs. (4–8) in the continuous domain, it is relatively straightforward to apply a discretization procedure to convert the problem into a discrete nonlinear-programming (NLP) problem. In general terms, for a level of discretization governed by the value of N the problem would be one of finding the vector of variables

$$\boldsymbol{X} = [\boldsymbol{x}(t_0), \boldsymbol{x}(t_1), \ldots, \boldsymbol{x}(t_N), \boldsymbol{u}(t_0), \boldsymbol{u}(t_1), \ldots, \boldsymbol{u}(t_N), T_p] \quad (10)$$

to minimize

$$\mathcal{J}_N = \sum_{i=0}^{N} \zeta_1\left\{\left\|\boldsymbol{w}_u^T[\boldsymbol{u}(t_i) - \boldsymbol{u}_d(t_i)]\right\|_q\right\} \quad (11)$$

subject to the constraints

$$\zeta_2\{\boldsymbol{f}[\boldsymbol{x}(t_i), \boldsymbol{u}(t_i), t_i; p]\} = 0, \qquad i = 0, \ldots, N \quad (12)$$

$$\boldsymbol{x}(t_0) - \boldsymbol{x}(t_N) = 0, \qquad \boldsymbol{u}(t_0) - \boldsymbol{u}(t_N) = 0 \quad (13)$$

$$\boldsymbol{g}_L \le \boldsymbol{g}[\boldsymbol{x}(t_i), \boldsymbol{u}(t_i), t_i; p] \le \boldsymbol{g}_U, \qquad i = 0, \ldots, N \quad (14)$$

where $\boldsymbol{\zeta}_1[\cdot]$ and $\boldsymbol{\zeta}_2[\cdot]$ denote functions that are dependent on the choice of discretization. A range of discretization methods is available. (See Ref. 21 for a discussion of some available methods and a comparison for standard optimal control problems.) In this Note, a discretization based on Simpson quadrature has been selected. The NLP is solved using the sequential-quadratic-programming software SNOPT (sparse nonlinear optimization).[22] SNOPT was originally developed in FORTRAN, but is called via MATLAB® using a mex-file interface.

Libration Control in Elliptical Orbits

The problem of controlling the tether librations in an elliptical orbit is an application of the preceding ideas, where the cost is selected as

$$\mathcal{J} = \int_0^{2\pi} u^2\,\mathrm{d}t \quad (15)$$

where u is the nondimensional reel acceleration. The period of the orbit is fixed according to the frequency entrainment law. Pelaez and Andres[20] demonstrated that periodic solutions for elliptical orbits are only possible for $T_p = 2\pi$ or 4π. For the purposes of this study, focus is only on the 2π solutions.

Feedback Control with Periodic Coefficients

Assuming that a controlled open-loop periodic trajectory has been determined, it is desirable to design a feedback controller to stabilize the motion. To analyze the stability of the controller, the form of the feedback controller is sought such that

$$\delta \boldsymbol{u} = \boldsymbol{K}(t)\delta \boldsymbol{x}(t) \tag{16}$$

where $\boldsymbol{K}(t) = \boldsymbol{K}(t + T_p)$ is the time-varying feedback gain matrix, which has periodicity equal to the period of the reference trajectory.

To fulfill these requirements, the feedback control law in this Note is designed to minimize the cost:

$$\delta J = \int_t^{t+T_p/2} [\delta \boldsymbol{x}^T(t^*)\boldsymbol{Q}\delta \boldsymbol{x}(t^*) + \delta \boldsymbol{u}^T(t^*)\boldsymbol{R}\delta \boldsymbol{u}(t^*)]\,\mathrm{d}t^* \tag{17}$$

subject to

$$\delta \boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_{\mathrm{ref}}(t), \qquad \delta \boldsymbol{x}(t + T_p/2) = 0 \tag{18}$$

where $\delta \boldsymbol{x} \in \mathbb{R}^{n_x}$ are the perturbed states, $\delta \boldsymbol{u} \in \mathbb{R}^{n_u}$ are the perturbed controls, $\boldsymbol{Q} \in \mathbb{R}^{n_x \times n_x}$ is a positive semidefinite weighting matrix, and $\boldsymbol{R} \in \mathbb{R}^{n_u \times n_u}$ is a positive-definite weighting matrix. This is a receding horizon control problem with terminal constraints. The feedback gain matrix is determined numerically using a discretization approach (see Refs. 23 and 24 for more details). If one substitutes $\boldsymbol{u}(t) = \boldsymbol{u}_{\mathrm{ref}}(t) + \boldsymbol{K}(t)\delta \boldsymbol{x}(t)$ into the governing equations, then the resulting set of equations is a nonlinear set of dynamic equations with periodic coefficients.

Stability of Solutions

The stability of the solutions can be analyzed using Floquet theory. According to this theory, the stability of the periodic orbit is governed by the eigenvalues λ of the monodromy matrix M. For the orbit to be stable, it is necessary that $|\lambda_i| < 1$, $i = 1, \ldots, 4$. If any eigenvalue has a magnitude greater than 1, then the solution is unstable. The computation of the results in this Note have been performed using numerical computations of the monodromy matrix.

Numerical Results

Numerical results were determined over a range of eccentricities from 0.05 through 0.90 in increments of 0.05 except in the region where the solutions transition from zero-control input periodic solutions to ones where reel actuation is required. Manual refinement of the orbit eccentricity was performed until a suitable level of precision was attained (five decimal places). The reel acceleration was limited in the range $-0.5 \le u \le 0.5$, which becomes active only for $e = 0.9$. There exists a family of periodic trajectories in the range $0 \le e \le e^*$, where $e^* \approx 0.44562$ for which the zero-control input solution is a feasible one. This value agrees with results using the continuation of periodic trajectories approach.[20] This particular family starts from the initial condition $\theta = 0$. Beyond the critical eccentricity value, there are no bounded periodic solutions that belong to the family. However, with the help of tether reeling, controlled periodic solutions are possible. Figure 1 illustrates the nature of the solutions in the $(\theta, \theta', \Lambda')$ phase space. It is evident that as the orbit eccentricity increases the tether reeling required to maintain the periodic orbit also increases.

The stability of the open-loop solutions was determined using Floquet theory. The magnitude of the eigenvalues of the monodromy matrix is shown in Fig. 2a. Note that two eigenvalues are shown, corresponding to the libration dynamics. The variation in tether length is assumed to be fixed according to the periodic control law, that is, Eq. (2) is not utilized in the calculations. The solutions are unstable inside the range $0.353 < e < 0.427$ (Ref. 20) and for $e > 0.5$. Outside this range, there appear to be some eccentricities for which the solutions are linearly stable, that is, where the modulus of the eigenvalues takes on the value 1. For $e > 0.5$, the modulus of the unstable eigenvalue increases to values on the order of 50.

Feedback gains were determined for state and control weightings defined by $\boldsymbol{Q} = q\boldsymbol{I}_{4\times 4}$ and $R = r$, respectively, for ratios $q/r = 0.1$,

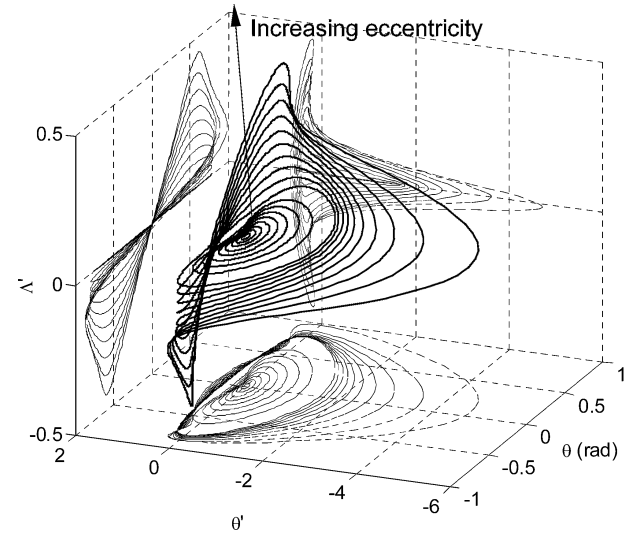

Fig. 1 Controlled periodic trajectories in $(\theta, \theta', \Lambda')$ phase space. (Two-dimensional phase space projections are shown on the walls of the plot.)

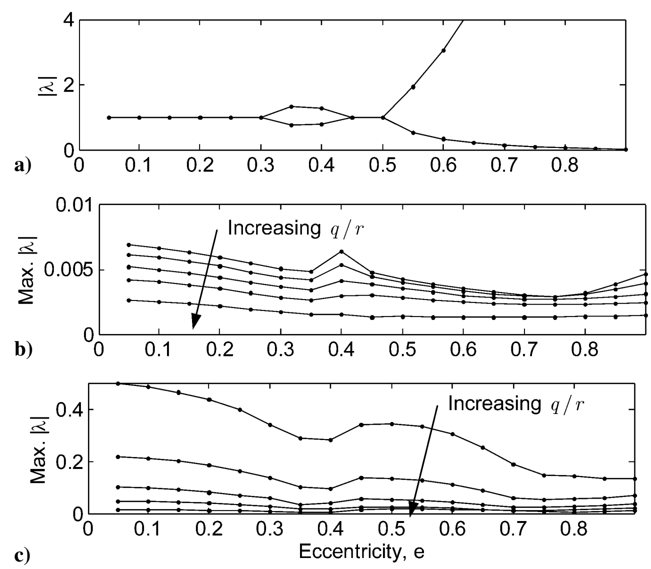

Fig. 2 Eigenvalues of the monodromy matrix for a) open-loop solution, b) closed loop with terminal constraints (q/r = 0.1, 0.3, 0.6, 1.0, 0.2), and c) closed loop without terminal constraints (q/r = 0.1, 0.3, 0.6, 1.0, 0.2).

0.3, 0.6, 1.0, 2.0. Cases with and without the terminal constraints were considered. The stability of the closed-loop solutions were assessed by applying Floquet theory to the closed-loop system. The maximum modulus of the eigenvalues of the monodromy matrix is shown in Fig. 2b, for the case with terminal constraints, and Fig. 2c for the case without terminal constraints. All of the closed-loop solutions are stable with the stability increasing with the ratio q/r. Furthermore, the stability is greatly enhanced through the use of terminal constraints.

Figure 3 shows a set of 15 perturbed trajectories in the $(\Delta\theta, \Delta\theta', \Delta\Lambda')$ phase space. The trajectories are for the case of $e = 0.65$ simulated for 10 orbits, with the initial conditions perturbed randomly in the range

$$\left|\Delta\theta_0'\right| = \left|\theta_0'\right| \tag{19}$$

which is a significant perturbation to the system. In all cases, the trajectory is attracted to the origin, demonstrating the excellent closed-loop behavior of the system. The tension is always positive, and the perturbations are damped in approximately one orbit. Hence, this approach might be more attractive than using thrusters for the same purpose.

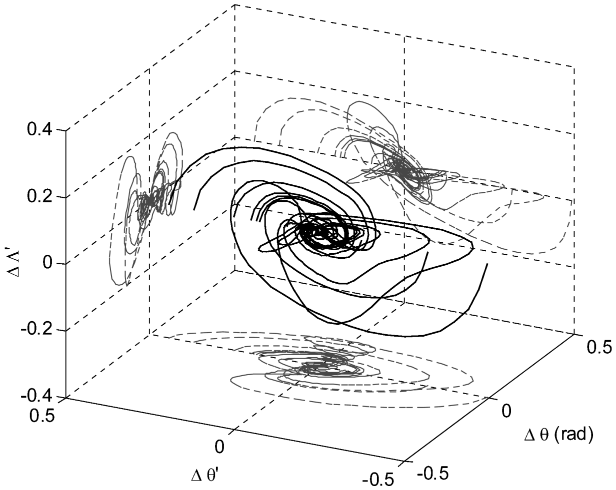

Fig. 3 Closed-loop responses in ($\Delta\theta$, $\Delta\theta'$, $\Delta\Lambda'$) phase space for $e = 0.65$. (Two-dimensional phase space projections are shown on the walls of the plot.)

Conclusions

A strategy for the control of the librations of a tethered satellite system in elliptic orbits using tether length control, which drives the system to controlled periodic libration trajectories, is suggested. There is a range of eccentricities up to about 0.4453 for which no length variations are needed for the system to follow the periodic trajectory. Above this eccentricity, it is necessary to vary the length of tether to maintain a periodic trajectory. The method for finding these trajectories to minimize the control input utilizes a collocation solution. Closed-loop stability is provided by a linear feedback control law, whose feedback gains are also periodic. Consequently, Floquet theory demonstrates the stability of the closed-loop system.

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